SOME PHILOSOPHICAL APPLICATIONS OF REFUTATION SYSTEMS

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- General proof-theory program. In 1974, Prawitz proposed to emancipate proof-theory from classical foundational studies by means of a new research program that he termed *general proof-theory.* The core of this program consists in the fact that "proofs are studied in their own right where one is interested in general questions about the nature and structure of proofs [...]" [10, p.66].

  The introduction of refutation calculi can be seen as a way to 'maximize' Prawitz's program inasmuch as they allow us to widen the space of proofs by including derivations ending with *invalid* formulas/sequents [21, 6, 17, 18]. Here the intuition is that one can have a better understanding of the structure of proofs once the 'positive' and the 'negative' parts are taken together as two alternative and complementary ways to syntactically characterize a given (decidable) logic.

  On the one hand, this idea echoes what happens in Girard's *Ludics* where proofs and para-proofs pacefully live and interact together [5]. On the other, it can be also seen as a way to further extend Wansing's proposal of providing logical hospitality to dual proofs [22]. Indeed, from an epistemic point of view, ontological parsimony is more an obstacle to the real understanding of the structure of proofs than a virtue.

- Meaning theory and proof-theoretic semantics. As a byproduct of the ontological extension supported in the previous point, refutation calculi may provide new conceptual tools in the fields of the anti-realist meaning theory and proof-theoretic semantics [3, 16]. Hardcore proof-theorists believe that the meaning of logical operators is primarily conveyed by their rules in a suitable proof-system. Semantics comes at a later time to provide a mathematical account of this very intuitions [20, 16].

Yet, this view has been discussed and criticized in several occasions; for instance, it is well-know the logico-philosophical debate about the *tonk* connective [11, 2, 1].

It is my conviction that several of the problems stemmed in this field might be fruitfully approached by considering the meaning of logical operators as provided by their rules in *both*, the positive and the negative parts. Take for instance the classical conjunction operator $\wedge$. According to this view, its meaning should be considered as provided not only by its rules in the positive part LK:

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \wedge\vdash \qquad \frac{\Gamma \vdash A, \Delta \qquad \Gamma' \vdash B, \Delta'}{\Gamma, \Gamma' \vdash A \wedge B, \Delta, \Delta'} \vdash\wedge,$$

but *also* by its rules in the negative part $\overline{\mathsf{LK}}$ [6]:

$$\frac{\Gamma, A, B \dashv \Delta}{\Gamma, A \wedge B \vdash \Delta} \wedge\dashv \quad \frac{\Gamma \dashv A, \Delta}{\Gamma \dashv A \wedge B, \Delta} \dashv\wedge \quad \frac{\Gamma \dashv B, \Delta}{\Gamma \dashv A \wedge B, \Delta} \dashv\wedge.$$

- Philosophy of logic. Consider for instance $\overline{\mathsf{LK}}$, the sequent system sound and complete with respect to the set of classically invalid sequents [6]. It seems quite reasonable to say that $\overline{\mathsf{LK}}$ is actually a *logic* to the extent that it provides an alternative syntactic characterization of propositional classical logic.

  Now, it is worth observing that, unlike LK, $\overline{\mathsf{LK}}$ is *paraconsistent*. In other words, classical logic can be syntactically grasped, albeit in the negative, by means of a paraconsistent sequent system. In general, any decidable logic whose semantics circumscribes a set of contingent formulas allow for a complementary characterization which is paraconsistent [12]. This kind of observations provide new insights about the logical nature of paraconsistency which turns out to be something sensitive to the specific syntactic formulation of our logic. Since $\overline{\mathsf{LK}}$ does not need to resort to stuctural rules, similar considerations also apply to the notion of *structurality* [13, 9].

- Rejection/negation debate. Is the negation of φ the same as rejecting φ? Notoriously, Frege maintained the idea that *rejection* and *negation* have not to be notationally distinguished as the act of rejecting a proposition φ is nothing else but the act of affirming its negation [4]. The opposite view is called *bilateralism* and finds in Smiley one of its more tenacious supporters [19, 7].

This debate kept alive and lively though the years involving both philosophers of language and logicians interested in investigating the nature of the negation operator [15, 8, 14, 7]. It would be interesting to understand how technical advances in the proof-theory of rejection calculi might contribute to elucidate this rejection/negation relation.

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